Global Dynamics of Connected Vehicle Systems

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Abstract

Road vehicles will vastly change in the coming years, due to the rise in autonomous vehicle technologies, sustainable forms of power, and better safety systems. This paper explores the improvements in traffic flow and safety that could be led by the shift to autonomously driven vehicles. Utilising VEHICLE-TO-VEHICLE (V2V) COMMUNICATION, it is possible to greatly reduce any time delays that arise due to human reaction times, or actuation delays. This model also takes into account any ACCELERATION SATURATIONS due to vehicle limitations. A model is developed that can work in an environment of both human-driven and connected autonomous vehicles (CAVs) - this lets us see how effective V2V communication is when in a HETEROGENEOUS environment, which represents the transition phase towards automated vehicle uptake (as automated vehicles are, at present, more expensive than 'traditional' vehicles) and performed a stability analysis to identify potential parameter values. We identified that effects could be seen in the global system from systems with 5% autonomous vehicles. This model is computationally simulated in Python and, along with stability analysis, shows that the addition of autonomous vehicles has the ability to reduce, and often eliminate, traffic waves when in a heterogeneous environment. The effects of additional obstacles, such as traffic lights and speed limits are also investigated. We observe the effects of these obstacles on the heterogeneous system's stability. The simulation is available as an open-source program that can be adapted to analyse other connected vehicle systems.

Introduction

As autonomous vehicles are introduced into the global vehicle system, they will have an effect on the traffic flow. Various reports predict different rates of autonomous vehicle uptake. In (1) predictions range from 20 - 70% uptake by 2050. Even the lower bound of 20% is a significant proportion and will have a noticeable effect on the global system. Design parameters of the autonomous and human vehicles can be changed in the manufacturing process. This paper investigates what we suggest these parameters should be set to, during and after the introduction of autonomous vehicles.

The behaviour of the global system is also seen to change when various non-linearities are introduced. For example, a heterogeneous system may act differently where the speed limit is higher, and the length of track is lower. The non-linearities we considered are: Speed limits (including VARIABLE SPEED LIMITS (VSL)), traffic lights, reaction time, acceleration saturation and the average headway (h^*). We investigate how these impact the traffic flow and safety of the system, comparing various systems with different proportions of autonomous and human vehicles.

Governing Equations

The governing equations for both autonomous (Equation 1) and human-driven vehicles (Equation 2) can be given as:

where $i = i_a$:

$$\dot{v}_i = f(\alpha(V(h_i^\sigma) - v_i^\sigma) + \sum_{j=1}^{k_i} \beta_j (v_{i+j}^\sigma - v_i^\sigma)) \tag{1}$$

where $i = i_h$:

$$\dot{v}_i = f(\alpha_h(V(h_i^\tau) - v_i^\tau) + \beta_h(v_{i+1}^\tau - v_i^\tau)) \tag{2}$$

The input for the governing equation is h, standing for headway, which is defined as the distance from the front of the vehicle to the back of the next vehicle. The governing equations model a system where vehicles are on a ring road, such as in the case shown in Figure 1, where there are 3 vehicles.

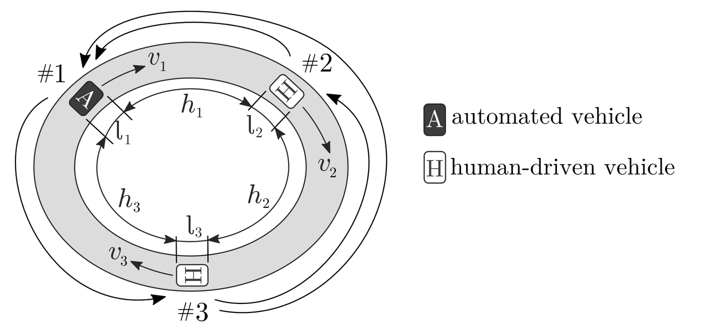

Figure 1: The governing equations model the interactions of vehicles on a ring road (2).

$V(h)$ is the OPTIMAL VELOCITY (OV) function, a curve representing the optimal velocity that is smooth at both h_{go} and h_{stop} and is plotted in Figure 2:

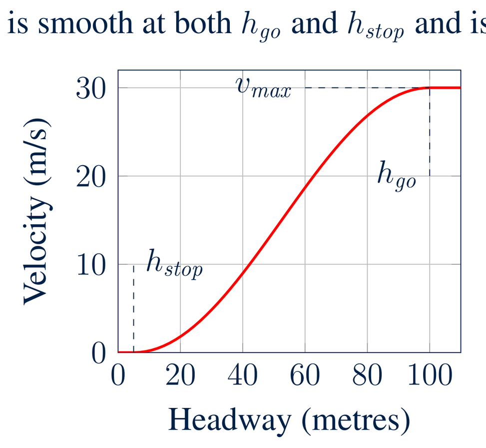

Figure 2: The optimal velocity (OV) function used in this paper, where h_{stop} = 5 metres, h_{go} = 100 metres and v_{max} = 30 m/s.

We propose that to understand the governing equations intuitively, it is beneficial to break it into two parts, the VELOCITY ERROR and the VELOCITY DELTA.

The velocity error is given by the following:

$$\alpha(V(h_i^\theta) - v_i^\theta)$$

$V(h)$ refers to the OV function, which gives the velocity the vehicle would ideally go at given the headway to the next vehicle (disregarding any delays or capabilities). The optimal velocity function used is from (2), which is based on real-world data. The methods used to collect the real-world data are explained in (3).

From this, we can see that the velocity error gives the *difference* between the ideal velocity and the velocity that the given vehicle i is going at now.

α (or α_h in the case of a human vehicle) has been described as a 'sensitivity' by Bando et al. in (4). This refers to the rate at which the vehicle will adjust to the OV, and is found by the reciprocal of the time it takes to adjust to the new velocity.

The velocity delta is given by the following:

where $i = i_a$:

$$\sum_{j=1}^{k_i} \beta_j (v_{i+j}^\sigma - v_i^\sigma)$$

The velocity delta part of the governing equation takes the sum of the differences in velocity between the autonomous vehicle i and all the human vehicles in front of it, which are indexed as $i + j$. Note that we do not look beyond the next autonomous vehicle. These are multiplied by the control parameter β_j. This parameter is directly related to the velocity delta and refers to the rate at which vehicle i will adjust to the sum of differences in velocity to the vehicles in front of itself. As the autonomous vehicle takes into account the velocities of all vehicles up to the next autonomous vehicle, this cascade of vehicles move smoothly as a group in the context of the entire system.

where $i = i_h$:

$$\beta_h(v_{i+1}^\tau - v_i^\tau)$$

We assume that human drivers only react to the velocity of the vehicle ahead of themselves, due to the vehicle in front blocking the line of sight between the driver and the vehicles further ahead. Thus a summation is not used for the instance where the driver is human.

Simulation

To make a simulation in Python of this model, we needed to slightly alter the equations to include time using Euler's method. We included a TIMESTEP (δ_t). The vehicles calculate their accelerations every δ_t. This number is set at a small value (0.02s in this simulation) which balances computational intensity and accuracy.

We were able to change parameters used within the governing equation (such as α_h and β_h), observing the effects they had on the stability of the system. The ring road model allowed the simulation of larger-scale dynamics (such as traffic waves) with the use of fewer vehicles. We also varied other parameters of the system, including the track length, number of vehicles, reaction times, acceleration saturation, and maximum velocity of each of the vehicles individually.

The scenario explored was 3 vehicles (two human, one autonomous) on a circular ring road, as in Figure 1 and as done in (2). In this case, the vehicles are further than h_{go} apart from one another, and therefore accelerate maximally, reaching their maximum velocity v_{max} without interfering with the velocities of any adjacent vehicles. Figure 3 plots the velocities of these three vehicles over time, showing that they stabilise at v_{max} within 20 seconds, with minimal oscillations of velocity.

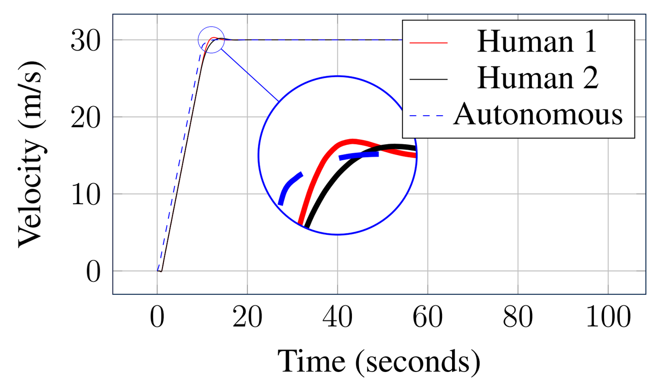

Figure 3: Three vehicles (one autonomous) on an arbitrarily long track. Here the vehicles are spaced apart enough to not affect the other vehicles' velocities. (Parameters: α = 2, α_h = 0.4, β = 0.2 and β_h = 0.2)

Note that in the magnified section of Figure 3, it can be seen that the human vehicles accelerate too much, speeding to above their v_{max}, before decelerating back to v_{max}. This is due to their larger reaction time in comparison to autonomous vehicles. Conversely, it can be seen that autonomous vehicles do not over-accelerate and smoothly reach the maximum velocity. This is important when traffic waves are considered, since the over/under acceleration of human-driven vehicles causes small velocity changes to amplify and create traffic waves, whereas this does not occur with autonomous vehicles.

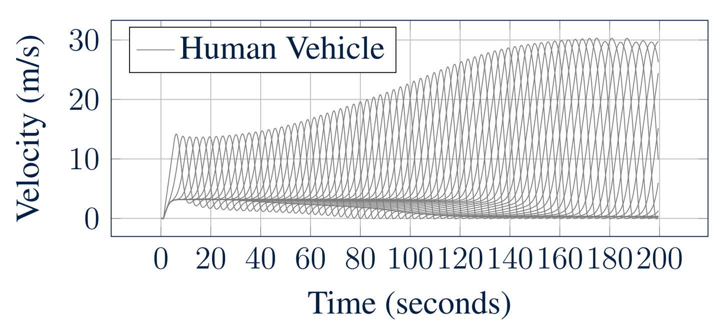

Figure 4: The formation of a traffic wave with 20 equally-spaced human-driven vehicles on a track of length 650 metres.

Replacing one of the human-driven vehicles with an autonomous vehicle reduces the amplitude of the traffic waves, as shown in Figure 5.

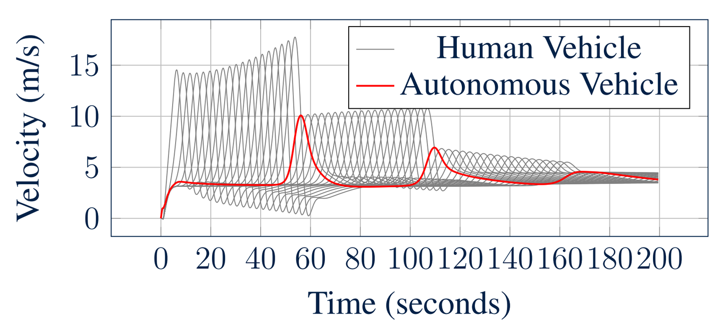

Figure 5: The replacement of one human vehicle with an autonomous vehicle eliminates the traffic wave shown in Figure 4.

The addition of one autonomous vehicle to the system of 20 human-driven vehicles allowed the amplifying effects of humans over-accelerating to be reduced. When the traffic wave reaches the autonomous vehicle, it smoothly accelerates, breaking the feedback loop. If the amplitude decrease by this effect is larger than the increase in amplitude by the compounding over-acceleration, the traffic wave can be eventually eliminated. This example suggests that even if there are only 5% of automatic vehicles in the system, traffic waves can potentially be mitigated. According to (1), this could be achieved optimistically by 2030 and pessimistically by 2040 in some countries.

Stability Analysis

Utilising the simulation we are able to make some estimates on the stability of non-linear systems by observing changes in the model made when parameters are changed. Key parameters include α_h, β_h, track length L, the number of vehicles N, cascade length and α. There are significant limitations to this method as only some key situations were analysed, but this nonetheless allowed us to observe and make some conclusions about the effects of a heterogeneous CVS in a non-linear system. The first heatmap represents a homogeneous system of human vehicles, while the second represents a system with 25% autonomous vehicles.

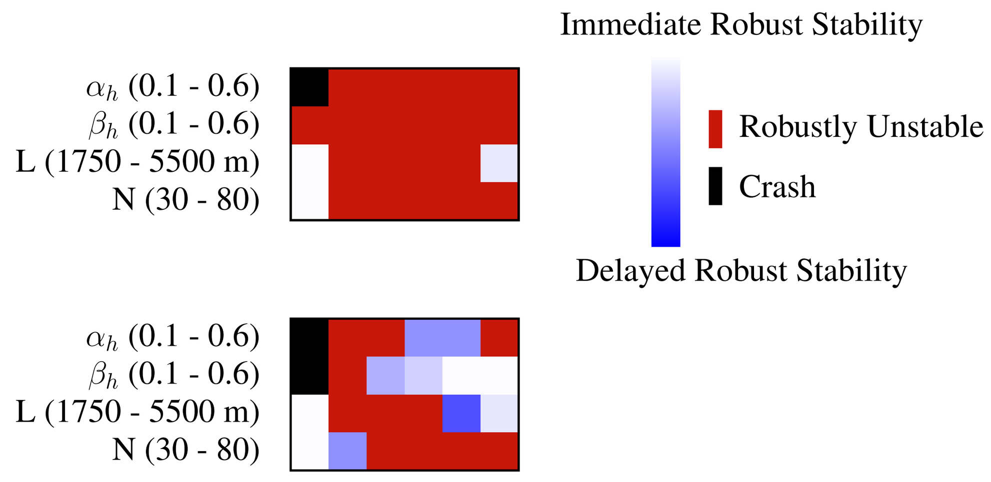

Figure 6: Discrete stability analysis performed on scenarios with varying parameters, showing how the percentage of autonomous vehicles present affects the stability of the system (when varying parameters).

A higher α_h of 0.4 - 0.5 allows the system to stabilise with a high percentage of autonomous vehicles, but for all other systems this system is unstable. When we observed the results, we found that the higher α_h causes more over-accelerations to happen in a shorter period, creating more traffic waves in systems with no or low percentage of autonomous vehicles. Conversely, in a system with a high percentage of autonomous vehicles, the higher α_h allows the autonomous vehicles to quickly dampen the traffic waves and achieve stability in the system. Note that with a too few autonomous cars, the attempt of autonomous cars to mitigate the traffic wave could sometimes result in more crashes when α_h was too low.

A higher β_h has a noticeably large effect on the stability of the system in heterogeneous systems. The higher β_h allows oscillations within each cascade to be dampened which results in significantly more stable systems. In (3), the realistic value of β_h was found to be 0.4 - 0.6. This works well with heterogeneous systems according to our model.

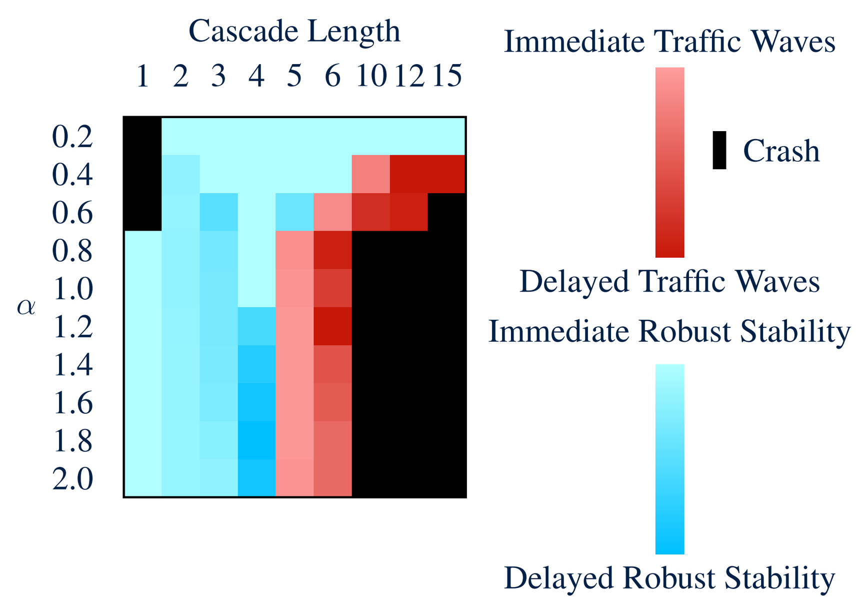

Figure 7: Discrete stability analysis performed on scenarios where cascade length and α are varied.

It is observed, in Figure 7, that low values of α result in the most stable connected vehicle systems. However, a lower α does not necessarily create a safer connected vehicle system. A lower α results in the velocity delta contributing more to the velocity of the vehicles which compounds quickly in a large cascade. This can create issues in a global system where other non-linearities that cause a deceleration of one of the vehicles in the cascade can affect the acceleration of the autonomous vehicle an unreasonable distance away. This is a limitation of the fundamental governing equations, and we believe the velocity delta should be adjusted in future reports to facilitate this.

Where the cascade length is large, high values of α result in crashes due to the limitations of the velocity delta as discussed. Lower α values still result in more stable systems than homogeneous systems, where the autonomous vehicles can move smoothly to dampen the traffic waves (shown by Figure 5). We hypothesise that the stability of the system is governed by an inversely proportional relationship between the cascade length and α.

Heterogeneous systems are shown to be ineffective when cascade length $\geq$ 10 and α is high. For large cascades, low values for α cause the connected vehicle system to be stable. Therefore, it is proposed that, during the initial phases of deployment of autonomous vehicles (when cascade lengths will be very high), the value of α should be set low (0.1 - 0.4).

Conclusion

In this paper, various CAVs were investigated. We found that autonomous vehicles can effectively dampen and eliminate traffic waves from as little as 5% of the system, given the correct parameter values. We used a discrete stability analysis to identify crucial parameters for various CAVs. We propose that CAVs with a cascade length of over 10 are relatively low impact but will have an improvement on the global system. For these CAVs, the α value should be set sufficiently low to overcome limitations in the velocity delta that is defined by the governing equations. We identified cascade lengths of 2 - 4 to be most robustly stable and suggest α values of approximately 0.8 - 1. We hypothesise that the ideal α value is inversely proportional to the cascade length. A high β_h is highly effective in CAVs with approximately 25% autonomous vehicles, as this helps the human vehicles within a cascade to effectively dampen the traffic waves. We also observed that in systems with a low percentage (around 10%) of autonomous vehicles, more crashes may occur when the α value is too high and $h_{stop} < h^* < h_{go}$. It has been shown that autonomous vehicles have a far greater effect in reducing the effects of traffic waves than existing traffic-mitigation methods, such as variable speed limits.

References

[1] Ahmadreza Talebian and Sabyasachee Mishra. Predicting the adoption of connected autonomous vehicles: A new approach based on the theory of diffusion of innovations. *Transportation Research Part C: Emerging Technologies*, 95:363–380, 2018.

[2] Adam K Kiss, Sergei S Avedisov, Daniel Bachrathy, and Gábor Orosz. On the global dynamics of connected vehicle systems. *Nonlinear Dynamics*, 96:1865–1877, 2019.

[3] Sergei S Avedisov, Gaurav Bansal, Adam K Kiss, and Gábor Orosz. Experimental verification platform for connected vehicle networks. In *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, pages 818–823. IEEE, 2018.

[4] Masako Bando, Katsuya Hasebe, Akihiro Nakayama, Akihiro Shibata, and Yuki Sugiyama. Dynamical model of traffic congestion and numerical simulation. *Physical review E*, 51(2):1035, 1995.